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Segmentation of the customers based on customer value: A three-way decision perspective

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ABSTRACT

This paper establishes an innovative value evaluation framework based on the criterion-oriented three-way decision (3WD) in the double hierarchy linguistic term (DHLT) environment to help the customer manager finish customer segmentation. Customer relationship management is the key to the success of enterprises in the information economy era. The segmentation of customers based on their relative criteria can identify the customers who are high-value customers for enterprises. However, multi-criteria decision-making can only display the value ranking of customers, rather than the value segmentation of customers. The employment of 3WD solves this problem. Then we classify the customers based on the 3WD method. First, the criteria are evaluated by using DHLTs, while the weights of criteria are acquired according to the maximum deviation method. Second, the conditional probabilities are estimated by the improved TOPSIS method combined with gray relation analysis, while the threshold values are calculated by the relative utilities which are constructed on the basis of the criterion information. Subsequently, the segmentation of customers is obtained according to the maximum-utility principle. Lastly, case research about the segmentation of customers based on value is used to demonstrate the practicality of our method, while some strategies about customer relationship management are given based on customer segmentation for obtaining maximum returns with minimum investment.

1. Introduction

Customer relationship management (CRM) is the key to the success of enterprises in the information economy era. Assessing customer value and classifying customers based on customer value is the core of CRM. The success of an enterprise depends on the enterprise's ability of building and maintaining loyal and valuable customer relationships. Thus, a refined strategy must be developed for customers according to their value [1]. Customer segmentation is an important means for enterprises to understand customers. Enterprises should first evaluate the value of customers, divide customers into different value categories, then formulate differentiated service strategies for customers so that enterprises can obtain maximum returns with minimum investment. Thus, various sophisticated data analysis techniques are applied to the segmentation of customers [2]. However, customer classification technology based on qualitative information is still lacking. Therefore, this paper will classify customers based on qualitative information.

For an enterprise, the value of its customers is evaluated. Then the customers are classified according to their value to identify the customers who have a high value. In this way, the enterprises can obtain maximum returns with minimum investment by formulating strategies of CRM. However, multi-criteria decision-making (MCDM) can displays the value ranking of customers, but it cannot present the value segmentation of customers. The three-way decision (3WD) method solves this problem. Then we categorize the value of customers by the 3WD method. The 3WD is decision-making method that conforms to the actual cognitive habits of human beings, which is summarized and refined by Yao [3] in the process of long-term research on decision-theoretic rough sets [4,5]. Recently, 3WD has been used in various fields, like employee recruitment [6], competent assessment [7], and work resumption [8]. To describe the qualitative characteristics of the real environment, 3WD has been extended into various fuzzy environments, like triangular fuzzy numbers [9], hesitant fuzzy sets [10], linguistic terms sets (LTSs) [11], uncertain linguistic terms [12], probabilistic linguistic term sets [13] and other tools of expression. To improve the accuracy of linguistic evaluation information [14], and make the evaluations of experts more flexible, the concept of double hierarchy linguistic term sets (DHLTSs) [15–17] was proposed. Then the customer manager can use the DHLTS to describe criterion information easily and quickly. Therefore, extending the 3WD into the DHLTS environment is essential.

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Conditional probabilities (CPs) and threshold values, as two essentials of 3WD, have been analyzed by many scholars [18,19]. The computation of conditional probability needs information on the decision criteria in traditional 3WD. But, the decision criteria are usually unknown in the employment of 3WD. Based on this motivation, Liang et al. [20] computed CPs by employing the TOPSIS method which avoids using the decision criteria. Since then, many multi-criteria integration methods [21,22] have been used to estimate CPs, such as gray relation analysis (GRA) [23], weighted aggregation (WA) [24], and by the comprehensive fuzzy concept value [25,26]. Jiang et al. [27] used positive gray incidence coefficient and negative gray incidence coefficient to construct CPs. Qi et al. [28] put forward conditional probability estimation strategies based on affinity propagation clustering algorithm under multiple criterion preferences. Tang and Qiao [29] used the hybrid distance to get the tolerance relation on the target set of an incomplete hybrid information system. Bisht and Pal [30] formed two states by the fuzzy c-mean clustering algorithm and constructed the similarity class for each alternative based on the Jaccard index. Inspired by these studies, we determined the CPs of the customers belonging to the high-value customers by the improved TOPSIS method combined with GRA. The relative proximity fully considers ideal distance [31] and gray relation coefficient [23], which is easy to be used and understood by the experts. While the criterion weight was determined by the maximum deviation method [32], which is one of the objective weighting methods. Jia and Liu [33] constructed the relative loss functions based on the multi-criteria environment, which can attain the threshold values objectively. Inspired by this study, we construct the relative utility functions to get the threshold values based on the maximum-utility principle [34]. The risk aversion coefficient is attained by criterion aspiration [35,36], which calculates threshold values more objectively.

We aim at establishing an innovative value evaluation framework based on the criterion-oriented 3WD in the DHLT environment to help customer management finish customer segmentation. Then the ultimate segmentation of customers is adopted to conduct CRM strategies for obtaining maximum returns with minimum investment. The specific motivations are described as follows: (1) DHLT environments are an innovative form of representation for describing 3WD problems. Then the experts can use the DHLTS to describe criterion information easily and quickly. Based on this phenomenon, this paper puts forward 3WD problems in DHLT environments. (2) The segmentation of customers based on their relative criteria can identify the customers who are high-value customers for enterprises. However, multi-criteria decision-making can only display the value ranking of customers, rather than the value segmentation of customers. The employment of 3WD solves this problem. Then we classify the customers based on the 3WD method.

To summarize, the contributions of the proposed method are shown below: (1) It uses the 3WDs method for the segmentation of the customers which is useful to make CRM strategies for obtaining maximum returns with minimum investment. (2) It tries to use DHLTs to characterize the qualitative criteria in 3WD which coincides with the expression ways of people. Then the customer managers can easily and quickly give DHLTs to describe criterion information, which can decrease classification cost and economize classification time. (3) It acquires the criteria weights according to the maximum-deviation method, and takes the relative proximity as the CPs which makes decisions more objective.

The prime arrangement of this paper is presented below: Section 2 characterizes the value evaluation criteria of the customers and displays the general construction of the solution. Section 3 reviews some basic concepts regarding DHLTS and 3WD, and shows a 3WD model for the segmentation of the customers in Section 4. Then, to verify the practicality of 3WD model, a case study about the segmentation of customers is given in Section 5. A sensitivity analysis of the parameter is displayed in Section 6. In Section 7, we compare our method with other methods and offer CRM strategies. Lastly, this research is concluded and the future research direction is elaborated.

2. Segmentation of the customers: System and solution

This section displays the value evaluation criteria of customers and shows the construction of the proposed solution.

2.1. Evaluation system of the customers

This research aim at evaluating the value of customers to calculate the CPs of the customers belonging to high-value customers, which are the essentials of the 3WD. Thus, the evaluation system of customer value need to be constructed first. The criteria are chosen below:

(1) Consumption ability (a_1). The consumption-ability of the customers is one of the important factors for enterprises to evaluate customers. After all, no matter how willing he is, if the person does not have the consumption ability, then he cannot create economic benefits for enterprises and cannot be counted as customer of the enterprises. Therefore, enterprises must know the consumption ability of customers. Consumption ability can be inferred from various sources and information.

(2) Loyalty degree (a_2). Loyalty degree is the degree of the customer's emotional preference for the enterprise or brand and the persistence of the enterprise or brand in behavior. Building customer loyalty is the most efficient method to reach persistent profit growth. Enterprises need to transform the concept of making transactions into the concept of establishing relationships with consumers, and change the focus from acquiring consumers to loyalty.

(3) Credit degree (a_3). Credit degree represents the credit performance of the customer during the transaction with the company. For example contract performance, and payment status. Of course, credit may also include the customer's social credit history. This indicator can be obtained from customer transaction records and related surveys.

(4) Brand communication value (a_4). Brand communication value is also called customer word-of-mouth value. The so-called word-of-mouth effect means that in the case of an efficient market and full flow of information, customers can influence the later purchasing patterns of current customers and future purchasing patterns of potential customers by spreading their perception of customer satisfaction and consumption experience. The customer's word-of-mouth value is related to the customer's purchase volume and loyalty, but it cannot be completely measured by the two, so it is listed as a single indicator here. Its effect value is mainly determined according to the scale, status, and influence of customers. This index can be obtained from market research data.

(5) Service cost (a_5). Service cost refers to all costs associated with customer service, including pre-sale, sale, and after-sales services. The lower the service cost required by the customer, the greater its contribution to the enterprise in terms of cost. In terms of cost contribution, old customers contribute more than previously unknown customers in this aspect. Old customers contribute more is because the cost of retaining old customers is lower than the cost of recognizing new customers, is also because old customers are more familiar with the use of products and reduce the requirements for service support of enterprises, which reduces the service cost of enterprises. This index can be evaluated based on historical empirical data.

We focus attention on conducting an integrated evaluation of the customer value based on the double hierarchy linguistic criterion information.

2.2. Framework of the offered solution

The offered solution conforms with the general process of 3WD. As displayed in Fig. 1, the steps of the offered solution are shown below:

Step 1: We collect the DHLT valuation about the criteria and the parameter from the customer manager. This initial information is the basis for 3WDs. The customer manager evaluates the criteria information based on the purchase history of the customers.

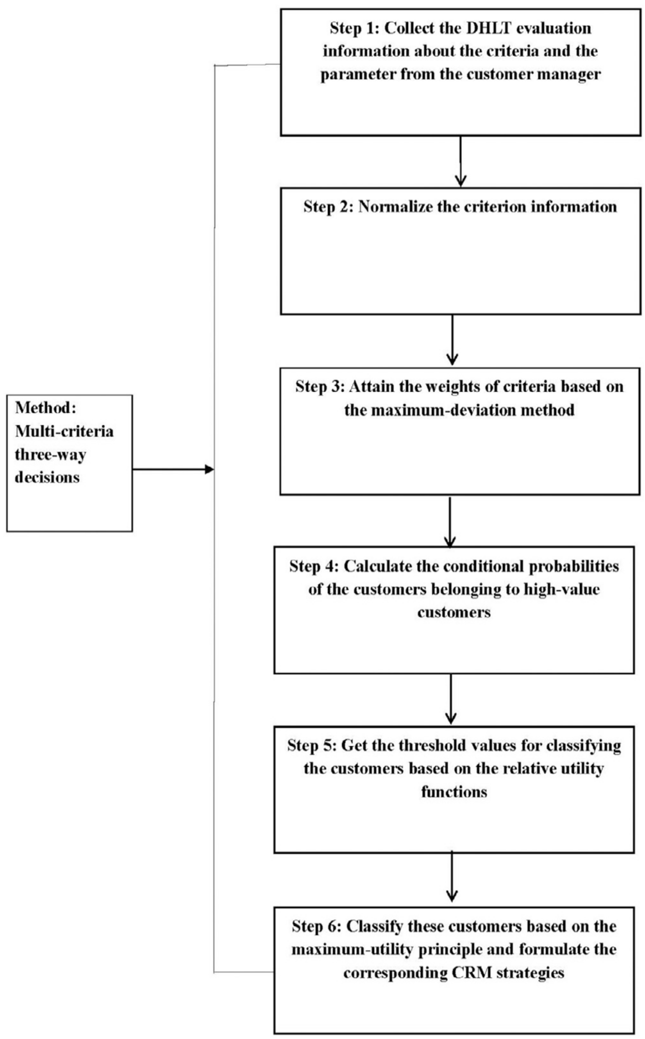

Fig. 1. The decision-making steps of our method.

Step 2: We normalize the criteria information. To help the calculation, we convert the DHLT information into real number information and the cost criteria into the benefit criteria.

Step 3: We attain the weights of criteria according to the maximum-deviation method. It maximizes the deviation of the criteria so as to make the difference of customers more obvious. Gaining weights based on criteria information makes decision-making more objective.

Step 4: We compute the CPs of the customers belonging to high-value customers. We calculate the relative proximity by the improved TOPSIS method combined with GRA and then attain the relative proximity as the conditional probability of each customer.

Step 5: We get the threshold values for classifying the customers based on the relative utility functions. We construct the relative cost and benefit functions with criteria aspirations and then calculate the relative utility.

Step 6: We classify these customers according to the maximum-utility principle and formulate the corresponding CRM strategies. The classification results are helpful to distribute scarce resources effectively, enhance the connection with customers, and obtain real competitive advantages.

3. Preliminaries

Some basic concepts of DHLTS are reviewed, and the 3WD is defined in this section.

3.1. DHLTSs

To describe the information more precisely, Gou et al. [15] defined the DHLTSs. DHLTS can be divided into two parts, the first hierarchy LTS is $S = \{s_p | p \in \{-\delta, \dots, -1, 0, 1, \dots, \delta\}\}$, the second hierarchy LTS $O = \{o_q | q \in \{-\varepsilon, \dots, -1, 0, 1, \dots, \varepsilon\}\}$ is used to embellish the first hierarchy LTS.

Definition 3.1 (*[15]*). Let S and O be the first and second hierarchy LTSs. Then, $S_O = \{s_{p\langle o_q\rangle} | p \in \{-\delta, \dots, -1, 0, 1, \dots, \delta\}; q \in \{-\varepsilon, \dots, -1, 0, 1, \dots, \varepsilon\}\}$ is a DHLTS, where $s_{p\langle o_q\rangle}$ represents the DHLT, s_p and o_q represents the first and the second hierarchy linguistic term.

It is worth noting that the order of the second hierarchy LTS must be exhibited according to the values of p [15]. When $p \geq 0$, the second hierarchy LTS needs to be selected with the ascending order. Otherwise, the second hierarchy LTS needs to be selected with the descending order. Specially, both s_δ and $s_{-\delta}$ only contain a half of area compared to other linguistic terms. Then we only use $O = \{o_q | q \in \{-\varepsilon, \dots, -1, 0\}\}$ and $O = \{o_q | q \in \{0, 1, \dots, \varepsilon\}\}$ to describe s_δ and $s_{-\delta}$, respectively. To manage the DHLT more simpler, two transformed functions are developed below:

Definition 3.2 (*[15]*). Given that $\overline{S}_O = \{s_{p\langle o_q\rangle}(c_i) | p \in [-\delta, \delta], q \in [-\varepsilon, \varepsilon], c_i \in U\}$ is a continuous DHLTS, $h_{\overline{S}_O} = \{s_{p\langle o_q\rangle}(c_i)\}$ is a double hierarchy linguistic element, and $h_\beta = \{\beta_i | \beta_i \in [0,1]\}$ is a set of numerical scales. For convenience, we substitute $s_{p_i\langle o_{q_i}\rangle}$ for $s_{p\langle o_q\rangle}(c_i)$. Two transformed functions g and g^{-1} between the numerical scale and the subscript $(p_i\langle o_{q_i}\rangle)$ of the DHLT $s_{p_i\langle o_{q_i}\rangle}$ are defined:

$$g : [-\delta, \delta] \times [-\varepsilon, \varepsilon] \to [0, 1],$$

$$g(p_i, q_i) = \frac{q_i + (\delta + p_i)\varepsilon}{2\varepsilon\delta} = \beta_i.$$

Then the transformation function G and G^{-1} between the DHLT $s_{p_i\langle o_{q_i}\rangle}$ and the numerical scale β_i can be formulated:

$$G : \overline{S}_O \to h_\beta, G(s_{p_i\langle o_{q_i}\rangle}) = g(p_i, q_i) = \frac{q_i + (\delta + p_i)\varepsilon}{2\varepsilon\delta} = \beta_i. \tag{1}$$

Given that $s_{p_1\langle o_{q_1}\rangle}$ and $s_{p_2\langle o_{q_2}\rangle}$ are two double hierarchy linguistic elements, when $G(s_{p_1\langle o_{q_1}\rangle}) > G(s_{p_2\langle o_{q_2}\rangle})$, then $s_{p_1\langle o_{q_1}\rangle} \succ s_{p_2\langle o_{q_2}\rangle}$; when $G(s_{p_1\langle o_{q_1}\rangle}) < G(s_{p_2\langle o_{q_2}\rangle})$, then $s_{p_1\langle o_{q_1}\rangle} \prec s_{p_2\langle o_{q_2}\rangle}$; when $G(s_{p_1\langle o_{q_1}\rangle}) = G(s_{p_2\langle o_{q_2}\rangle})$, then $s_{p_1\langle o_{q_1}\rangle} \approx s_{p_2\langle o_{q_2}\rangle}$.

3.2. 3WD

To improve the fault tolerance of decisions, Yao et al. [4,5] extended the two-way decision to the 3WD. To adapt to various complex situations, the 3WD is extended to various fuzzy environments. At present, some basic concepts of 3WD are defined under the DHLT environment first. The newly proposed 3WD model is consisting of two states and three behaviors. $\Omega = \{X, X^c\}$ represents an object that pertains to the state X or does not pertain to the state X. $Pr(X|c_i)$ and $Pr(X^c|c_i)$ denote CPs of an object c_i pertaining to the states X and X^c, respectively. The CPs are real numbers, and they have $Pr(X|c_i) + Pr(X^c|c_i) = 1$. As for the behavior set, $\Lambda = \{a_A, a_D, a_R\}$ implies the behavior of accepting, delaying, and rejecting adopted by the decision-maker, and a_A signifies $c_i \epsilon POS(X)$, a_D signifies $c_i \epsilon BND(X)$, and a_R signifies $c_i \epsilon NEG(X)$, where $POS(X), BND(X), NEG(X)$ denote the positive region, the boundary region, and the negative region of rough sets respectively. The states represent the global situation of the object, while the behaviors represent the comments of the decision-maker in the 3WD model. ϕ, φ, χ are the threshold values of 3WD. Then the decision-maker will provide the loss functions under different states based on their experience. Then the decision results will be derived according to the minimum-loss principle:

(P) If $Pr(X|c_i) \geq \phi$ and $Pr(X|c_i) \geq \chi$, imply $c_i \in POS(X)$;
(B) If $Pr(X|c_i) \leq \phi$ and $Pr(X|c_i) \geq \varphi$, imply $c_i \in BND(X)$; and
(N) If $Pr(X|c_i) \leq \varphi$ and $Pr(X|c_i) \leq \chi$, imply $c_i \in NEG(X)$.

4. 3WD model for segmentation of the customers

The value estimation structure for segmentation of the customers can be characterized below: Let $C = \{c_1, c_2, \ldots, c_m\}$ and $AT = \{a_1, a_2, \ldots, a_n\}$ denote the set of customers and evaluation criteria, separately. $V = \bigcup_{a \in AT} V_a$, V_a denotes a value domain of the criterion a. $e : C \times AT \to V$ denotes a function, such that $e_{ij} = e(c_i, a_j) \in V_a$ for every $c_i \in C, a_j \in AT$, where $e_{ij} = s_{p_{ij}\langle o_{q_{ij}}\rangle}$ denotes a DHLT, representing the estimation of the customer c_i about the criterion a_j. $u = (u_1, u_2, \ldots, u_n)^T$ represents the weight vector of all criteria. It satisfies: $0 \leq u_j \leq 1$ and $\sum_{j=1}^n u_j = 1$. If all criteria are positively correlated with the state X, $\widetilde{e}_{ij} = G(e_{ij})$ is the normalized information. If some criteria are negatively correlated with the state X, we normalize the information for these criteria through the formula $\widetilde{e}_{ij} = 1 - G(e_{ij})$. Next, a 3WD model will be put forward to attain the segmentation of the customers.

4.1. Weighting criteria based on maximum-deviation method

The acquisition of criterion weight is an important procedure in the MCDM process. If the difference of criterion values of all schemes under criteria is smaller, it indicates that criterion plays a smaller role in scheme decisions. Conversely, the criterion is important. Therefore, the criteria with larger deviation values of the scheme should be given greater weight. For the criterion a_j, we can calculate the deviation of the object c_i to the other objects as follows:

$$F_{ij}(u) = u_j \sum_{k=1}^{m} |G(\widetilde{e}_{ij}) - G(\widetilde{e}_{kj})|. \tag{2}$$

Then, the deviation of all objects to the other objects for the criterion a_j is:

$$F_j(u) = \sum_{i=1}^{m} F_{ij}(u) = u_j \sum_{i=1}^{m}\sum_{k=1}^{m} |G(\widetilde{e}_{ij}) - G(\widetilde{e}_{kj})|. \tag{3}$$

Based on the results reported in [32], we maximize all deviation values for all the criteria for determining the weight vector. The optimization model can be constructed as follows:

$$\max F(u) = \sum_{j=1}^{n} F_j(u) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} u_j |G(\widetilde{e}_{ij}) - G(\widetilde{e}_{kj})|, \tag{4}$$

$$s.t. \begin{cases} \sum_{j=1}^n u_j^2 = 1, \\ u_j \geq 0, j = 1, 2, \ldots, n. \end{cases}$$

Based on the results of [32], we further obtain the optimal solution as follows:

$$u_j = \frac{\sum_{i=1}^m \sum_{k=1}^m |G(\widetilde{e}_{ij}) - G(\widetilde{e}_{kj})|}{\sum_{j=1}^n \sum_{i=1}^m \sum_{k=1}^m |G(\widetilde{e}_{ij}) - G(\widetilde{e}_{kj})|}. \tag{5}$$

Then we can acquire the criterion weight based on the formula (5).

4.2. Calculating CPs

Many scholars have studied the CPs and the threshold values, which are two crucial elements in the 3WD models. The calculation of conditional probability is related to the decision criterion. But, the decision criteria are usually unknown in the employment of 3WD. Therefore, a new way needs to be proposed to calculate CPs. Liang et al. [20] estimated the CPs by employing the TOPSIS method. Li et al. [23] calculated the CPs according to the GRA method. Motivated by this, we apply the improved TOPSIS method combined with GRA to calculate the CPs. The CPs are determined according to the criterion information.

The distance between two customers c_i and c_k is defined as follows:

$$d(c_i, c_k) = \sum_{j=1}^{n} u_j |G(\widetilde{e}_{ij}) - G(\widetilde{e}_{kj})|. \tag{6}$$

The distance between the customer c_i and the relative positive ideal solution c^+ is calculated as follows:

$$D_i^+ = d(c_i, c^+). \tag{7}$$

The distance between the customer c_i and the relative negative ideal solution c^- is calculated as follows:

$$D_i^- = d(c_i, c^-). \tag{8}$$

Among them, $c^+ = \{\max_{1\leq i\leq m} \widetilde{e}_{i1}, \max_{1\leq i\leq m} \widetilde{e}_{i2}, \ldots, \max_{1\leq i\leq m} \widetilde{e}_{in}\}$, $c^- = \{\min_{1\leq i\leq m} \widetilde{e}_{i1}, \min_{1\leq i\leq m} \widetilde{e}_{i2}, \ldots, \min_{1\leq i\leq m} \widetilde{e}_{in}\}$.

The gray relation coefficient (GRC) between c_i and the relative positive ideal solution c^+ on the jth attribute is

$$r_{ij}^+ = \frac{\min_{1\leq i\leq m}\min_{1\leq j\leq n} \Delta_{ij}^+ + \xi \max_{1\leq i\leq m}\max_{1\leq j\leq n} \Delta_{ij}^+}{\Delta_{ij}^+ + \xi \max_{1\leq i\leq m}\max_{1\leq j\leq n} \Delta_{ij}^+}, \tag{9}$$

where $\Delta_{ij}^+ = c_j^+ - \widetilde{e}_{ij}$, for $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n, \xi \in [0, 1]$. ξ denotes the distinguishing coefficient. Normally, $\xi = 0.5$.

The GRC between c_i and the relative positive ideal solution c^+ is

$$R_i^+ = \sum_{j=1}^{n} w_j r_{ij}^+, i = 1, 2, \ldots, m. \tag{10}$$

The GRC between c_i and the relative negative ideal solution c^- on the jth attribute is

$$r_{ij}^- = \frac{\min_{1\leq i\leq m}\min_{1\leq j\leq n} \Delta_{ij}^- + \xi \max_{1\leq i\leq m}\max_{1\leq j\leq n} \Delta_{ij}^-}{\Delta_{ij}^- + \xi \max_{1\leq i\leq m}\max_{1\leq j\leq n} \Delta_{ij}^-}, \tag{11}$$

where $\Delta_{ij}^- = \widetilde{e}_{ij} - c_j^-$, for $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n, \xi = 0.5$.

The GRC between x_i and the relative negative ideal solution c^- is

$$R_i^- = \sum_{j=1}^{n} w_j r_{ij}^-, i = 1, 2, \ldots, m. \tag{12}$$

The positive proximity of the object c_i is calculated as follows:

$$K_i^+ = \alpha D_i^- + (1 - \alpha) R_i^+, \tag{13}$$

The negative proximity of the object c_i is calculated as follows:

$$K_i^- = \alpha D_i^+ + (1 - \alpha) R_i^-, \tag{14}$$

the parameter α represents the customer manager's preference degree of the distance measure over the GRC, and $\alpha \in [0, 1]$.

The relative proximity $X(c_i)$ is calculated by the positive proximity and the negative proximity. Then the conditional probability $Pr(X|c_i)$ is estimated through the relative proximity.

$$Pr(X|c_i) = X(c_i) = \frac{K_i^+}{K_i^+ + K_i^-}, \tag{15}$$

Proposition 4.1. *When $D_i^- R_i^- > D_i^+ R_i^+$, $Pr(X|c_i)$ is monotonically increasing with the increase of the parameter value α; when $D_i^- R_i^- < D_i^+ R_i^+$, $Pr(X|c_i)$ is monotonically decreasing with the increase of the parameter value α; when $D_i^- R_i^- = D_i^+ R_i^+$, $Pr(X|c_i)$ is invariable with the increase of the parameter value α.*

Table 1
The relative cost functions of the object c_i.

	$X(P)$	$X^c(N)$
a_A	$\lambda_{AP}=0$	$\lambda_{AN}=X_{\max}-X(c_i)$
a_D	$\lambda_{DP}=\eta_i(X(c_i)-X_{\min})$	$\lambda_{DN}=\eta_i(X_{\max}-X(c_i))$
a_R	$\lambda_{RP}=X(c_i)-X_{\min}$	$\lambda_{RN}=0$

Table 2
The relative revenue functions of the object c_i.

	$X(P)$	$X^c(N)$
a_A	$v_{AP}=X(c_i)-X_{\min}$	$v_{AN}=0$
a_D	$v_{DP}=(1-\eta_i)(X(c_i)-X_{\min})$	$v_{DN}=(1-\eta_i)(X_{\max}-X(c_i))$
a_R	$v_{RP}=0$	$v_{RN}=X_{\max}-X(c_i)$

Proof. $Pr(X|c_i)=\frac{K_i^+}{K_i^++K_i^-}=\frac{\alpha D_i^-+(1-\alpha)R_i^+}{\alpha D_i^-+(1-\alpha)R_i^++\alpha D_i^++(1-\alpha)R_i^-}=\frac{\alpha(D_i^--R_i^+)+R_i^+}{\alpha(D_i^-+D_i^+-R_i^+-R_i^-)+R_i^++R_i^-}$, because $\frac{\partial Pr(X|c_i)}{\partial\alpha}=\frac{D_i^-R_i^--D_i^+R_i^+}{(\alpha(D_i^-+D_i^+-R_i^+-R_i^-)+R_i^++R_i^-)^2}$, when $D_i^-R_i^->D_i^+R_i^+$, $\frac{\partial Pr(X|c_i)}{\partial\alpha}>0$, when $D_i^-R_i^-<D_i^+R_i^+$, $\frac{\partial Pr(X|c_i)}{\partial\alpha}<0$, when $D_i^-R_i^-=D_i^+R_i^+$, $\frac{\partial Pr(X|c_i)}{\partial\alpha}=0$. □

4.3. Calculating threshold values

In the traditional TWDs model, the threshold values are calculated based on the loss functions given by the DM. Jia and Liu [33] attached the loss functions to the multi-criteria information system which makes the acquirement of loss functions more objective. The relative cost functions $\lambda_{\cdot\diamond}(\cdot=A,D,R;\diamond=P,N)$ is attained as Table 1.

Among them, $X(c_i)\in[0,1]$, $X_{\min}=\min\{X(c_i)\}=0, X_{\max}=\max\{X(c_i)\}=1$.

The relative revenue functions $v_{\cdot\diamond}$ is attained as Table 2.

The relative utility functions $\pi_{\cdot\diamond}$ is attained according to the relative cost functions and the relative revenue functions as Table 3.

Among them, the risk aversion coefficient η_i is calculated as follows:

$$\begin{aligned}\eta_i&=\frac{|X(c_i)-X_{mid}|}{\max(|X_{\min}-X_{mid}|,|X_{\max}-X_{mid}|)}\\&=\frac{|X(c_i)-(X_{\max}+X_{\min})/2|}{\max(|(X_{\min}-X_{\max})/2|,|(X_{\max}-X_{\min})/2|)}\\&=\frac{|X(c_i)-(X_{\max}+X_{\min})/2|}{|(X_{\min}-X_{\max})/2|}=|2X(c_i)-1|.\end{aligned}\tag{16}$$

The risk aversion coefficient η_i is attained by criterion aspiration [35]. The closer the satisfaction degree of the criterion value is to the midpoint, the smaller the cost of the delay action a_D should be, and the smaller the value of η_i should be.

The expected utility $EU(a_\cdot|c_i)(\cdot=A,D,R)$ is calculated as follows:

$$EU(a_A|c_i)=\pi_{AP}Pr(X|c_i)+\pi_{AN}Pr(X^c|c_i);\tag{17}$$

$$EU(a_D|c_i)=\pi_{DP}Pr(X|c_i)+\pi_{DN}Pr(X^c|c_i);\tag{18}$$

$$EU(a_R|c_i)=\pi_{RP}Pr(X|c_i)+\pi_{RN}Pr(X^c|c_i).\tag{19}$$

Next, the maximum-utility classification rules can be attained below:

(P_1) If $EU(a_A|c_i)\ge EU(a_D|c_i)$ and $EU(a_A|c_i)\ge EU(a_R|c_i)$, decide $c_i\in POS(X)$;

(B_1) If $EU(a_D|c_i)\ge EU(a_A|c_i)$ and $EU(a_D|c_i)\ge EU(a_R|c_i)$, decide $c_i\in BND(X)$; and

(N_1) If $EU(a_R|c_i)\ge EU(a_A|c_i)$ and $EU(a_R|c_i)\ge EU(a_D|c_i)$, decide $c_i\in NEG(X)$.

Then we calculate the threshold values ϕ_i,φ_i,χ_i by Eqs. (20)–(22). Lastly, we categorize these customers based on the classification rules P_2-N_2.

(P_2) If $Pr(X|c_i)\ge\phi_i$ and $Pr(X|c_i)\ge\chi_i$, imply $c_i\in POS(X)$;
(B_2) If $Pr(X|c_i)\le\phi_i$ and $Pr(X|c_i)\ge\varphi_i$, imply $c_i\in BND(X)$; and
(N_2) If $Pr(X|c_i)\le\varphi_i$ and $Pr(X|c_i)\le\chi_i$, imply $c_i\in NEG(X)$.

Among them, the thresholds ϕ_i,φ_i,χ_i are determined by the relative utility $\pi_{\cdot\diamond},(\cdot=A,D,R;\diamond=P,N)$ as follows:

$$\begin{aligned}\phi_i&=\frac{\pi_{AN}-\pi_{DN}}{\pi_{AN}-\pi_{DN}+\pi_{DP}-\pi_{AP}}\\&=\frac{(1-\eta_i)(X_{\max}-X(c_i))}{(1-\eta_i)(X_{\max}-X(c_i))+\eta_i(X(c_i)-X_{\min})},\end{aligned}\tag{20}$$

$$\begin{aligned}\varphi_i&=\frac{\pi_{DN}-\pi_{RN}}{\pi_{DN}-\pi_{RN}+\pi_{RP}-\pi_{DP}}\\&=\frac{\eta_i(X_{\max}-X(c_i))}{\eta_i(X_{\max}-X(c_i))+(1-\eta_i)(X(c_i)-X_{\min})},\end{aligned}\tag{21}$$

$$\chi_i=\frac{\pi_{AN}-\pi_{RN}}{\pi_{AN}-\pi_{RN}+\pi_{RP}-\pi_{AP}}=\frac{X_{\max}-X(c_i)}{(X_{\max}-X(c_i))+(X(c_i)-X_{\min})}.\tag{22}$$

5. A case study about the segmentation of the customers

With the development of big data era and digital economy, a large amount of data information is generated, and the market competition among enterprises is fierce, so the attractiveness and competitiveness of enterprises to customers are essential to the development of enterprises. The key for enterprises to successfully acquire customers is determining the target customers first. Reasonable customer segmentation is the precondition for enterprises to improve their relationship with customers. The reasonable segmentation of customers helps enterprises to distribute scarce resources availably, enhance the connection with customers, and obtain realistic competitive superiorities. Then an enterprise that sells luxury bags decided to categorize a group of its customers based on the solution framework in Section 2.2.

Step 1: A group of customers needs to be classified according to their value. The customers are c_1-c_8. Before classifying these customers, the customer manager needs to estimate the value of these customers. In the frame of 3WD, the state set $\Omega=\{X,X^c\}$ denotes whether this customer belongs to the high-value customer or not. The value of customers is estimated according to the DHLT criteria information. Then the criteria are displayed as a_1: consumption ability; a_2: loyalty degree; a_3: credit degree; a_4: brand communication value; a_5: service cost. Then we employ the steps of Section 2.2 to conduct the research and categorize these customers.

To summarize, we can obtain: $C=\{c_1,c_2,\ldots,c_8\}$ and $AT=\{a_1,a_2,\ldots,a_5\}$. The customer manager adopts these two LTSs

$$\begin{aligned}S=\{&s_{-4}=extremely\,low,s_{-3}=very\,low,s_{-2}=low,\\&s_{-1}=slightly\,low,s_0=medium,\\&s_1=slightly\,high,s_2=high,s_3=very\,high,s_4=extremely\,high\}\end{aligned}$$

$$O=\{o_{-4}=far\,from,o_{-3}=scarcely,o_{-2}=only\,a\,little,o_{-1}=a\,little,$$

Table 3
The relative utility functions of the object c_i.

	$X(P)$	$X^c(N)$
a_A	$\pi_{AP}=v_{AP}-\lambda_{AP}=X(c_i)-X_{\min}$	$\pi_{AN}=v_{AN}-\lambda_{AN}=X(c_i)-X_{\max}$
a_D	$\pi_{DP}=v_{DP}-\lambda_{DP}=(1-2\eta_i)(X(c_i)-X_{\min})$	$\pi_{DN}=v_{DN}-\lambda_{DN}=(1-2\eta_i)(X_{\max}-X(c_i))$
a_R	$\pi_{RP}=v_{RP}-\lambda_{RP}=X_{\min}-X(c_i)$	$\pi_{RN}=v_{RN}-\lambda_{RN}=X_{\max}-X(c_i)$

Table 4
The DHLT information evaluation of customers provided by customer managers.

	a_1	a_2	a_3	a_4	a_5
c_1	$s_{0\langle o_2\rangle}$	$s_{-1\langle o_{-1}\rangle}$	$s_{1\langle o_1\rangle}$	$s_{0\langle o_{-2}\rangle}$	$s_{1\langle o_1\rangle}$
c_2	$s_{-2\langle o_{-2}\rangle}$	$s_{-3\langle o_0\rangle}$	$s_{-2\langle o_{-2}\rangle}$	$s_{-3\langle o_{-1}\rangle}$	$s_{2\langle o_2\rangle}$
c_3	$s_{2\langle o_3\rangle}$	$s_{3\langle o_3\rangle}$	$s_{2\langle o_0\rangle}$	$s_{3\langle o_2\rangle}$	$s_{-2\langle o_1\rangle}$
c_4	$s_{1\langle o_1\rangle}$	$s_{0\langle o_2\rangle}$	$s_{-1\langle o_2\rangle}$	$s_{0\langle o_2\rangle}$	$s_{-1\langle o_2\rangle}$
c_5	$s_{2\langle o_2\rangle}$	$s_{3\langle o_0\rangle}$	$s_{3\langle o_3\rangle}$	$s_{2\langle o_2\rangle}$	$s_{-3\langle o_{-1}\rangle}$
c_6	$s_{-2\langle o_1\rangle}$	$s_{-2\langle o_{-1}\rangle}$	$s_{-3\langle o_{-2}\rangle}$	$s_{-2\langle o_0\rangle}$	$s_{1\langle o_3\rangle}$
c_7	$s_{-3\langle o_2\rangle}$	$s_{-2\langle o_1\rangle}$	$s_{-1\langle o_{-2}\rangle}$	$s_{-2\langle o_{-2}\rangle}$	$s_{3\langle o_0\rangle}$
c_8	$s_{3\langle o_2\rangle}$	$s_{2\langle o_3\rangle}$	$s_{2\langle o_1\rangle}$	$s_{3\langle o_0\rangle}$	$s_{-2\langle o_{-2}\rangle}$

Table 5
The normalized criterion information.

	a_1	a_2	a_3	a_4	a_5
c_1	0.5625	0.34375	0.65625	0.4375	0.34375
c_2	0.3125	0.1250	0.1875	0.09375	0.1875
c_3	0.84375	0.96875	0.7500	0.9375	0.71875
c_4	0.65625	0.5625	0.4375	0.5625	0.5625
c_5	0.8125	0.875	0.96875	0.8125	0.90625
c_6	0.28125	0.21875	0.0625	0.25	0.28125
c_7	0.1875	0.28125	0.3125	0.1875	0.1250
c_8	0.9375	0.84375	0.78125	0.875	0.8125

Table 6
The relative proximity of each customer.

	D_i^+	D_i^-	R_i^+	R_i^-	K_i^+	K_i^-	$X(c_i)$
c_1	0.4769	0.3506	0.5116	0.5793	0.4311	0.5281	0.4494
c_2	0.7671	0.0604	0.3888	0.8930	0.2246	0.8300	0.2130
c_3	0.0979	0.7296	0.8531	0.3860	0.7914	0.2419	0.7659
c_4	0.3905	0.4370	0.5584	0.5099	0.4977	0.4502	0.5251
c_5	0.0689	0.7586	0.8852	0.3760	0.8219	0.2224	0.7870
c_6	0.7278	0.0997	0.4021	0.8295	0.2509	0.7787	0.2437
c_7	0.7236	0.1038	0.4016	0.8371	0.2527	0.7804	0.2446
c_8	0.0958	0.7317	0.8446	0.3827	0.7881	0.2393	0.7671

$o_0 = just\,right,$

$o_1 = much, o_2 = very\,much, o_3 = extremely\,much, o_4 = entirely\}$

to evaluate the criteria of the customers.

Then, we gather evaluations for each customer about the criteria $a_1 - a_5$ from the customer managers. They evaluate criteria according to the purchase history of customers and use DHLT to express evaluation information. The corresponding DHLT evaluation information is displayed in Table 4.

Step 2: We can find those criteria $a_1 - a_4$ are positively correlated with the state X, the criterion a_5 is negatively correlated with the state X. Then the normalized criterion information is shown as Table 5.

Step 3: Based on the maximum deviation method mentioned in Section 4.1, the weight vector of criteria can be attained: $u = (0.1826, 0.2098, 0.2051, 0.2110, 0.1915)$.

Step 4: Determine the optimal solution $c^+ = \{s_{3\langle o_2\rangle}, s_{3\langle o_3\rangle}, s_{3\langle o_3\rangle}, s_{3\langle o_2\rangle}, s_{3\langle o_0\rangle}\}$, the worst solution $c^- = \{s_{-3\langle o_2\rangle}, s_{-3\langle o_0\rangle}, s_{-3\langle o_{-2}\rangle}, s_{-3\langle o_{-1}\rangle}, s_{-3\langle o_0\rangle}\}$. The parameter α is set as 0.5. The relative proximity of each customer is calculated based on the improved TOPSIS proposed in Section 4.2, and the results are shown in Table 6.

Based on the relative proximity, we determine the conditional probability of each customer as Table 7.

The CPs are shown as $Pr(X|c_1) = 0.4494$, $Pr(X|c_2) = 0.2130$, $Pr(X|c_3) = 0.7659$, $Pr(X|c_4) = 0.5251$, $Pr(X|c_5) = 0.7870$, $Pr(X|c_6) = 0.2437$, $Pr(X|c_7) = 0.2446$, $Pr(X|c_8) = 0.7671$.

Step 5: In accordance with the evaluation information, the risk aversion coefficient η_i can be calculated by Eq. (16), and the thresholds $\phi_i, \varphi_i, \chi_i$ can be obtained by Eqs. (20)–(22). The outcome is shown as Table 8.

Step 6: According to the classification rules $P_1 - N_1$, we can attain the classification result of each customer. The results are shown below: $POS(X) = \{c_5, c_8, c_3\}$; $BND(X) = \{c_4, c_1\}$; $NEG(X) = \{c_7, c_6, c_2\}$. The customers c_5, c_8, and c_3 are categorized as high-value customers. The customers c_7, c_6, c_2 are categorized as low-value customers. The customers c_4, c_1 are required to acquire more information and wait for further segmentation. Based on the CRM strategy, for high-value customers, such as c_5, c_8, c_3, the enterprise should concentrate resources to focus on maintaining these customers, and try to maintain their loyalty to the company with quality service. For low-value customers, such as c_7, c_6, c_2, the enterprise does not need to invest too many resources to maintain. For the other customers located in $BND(X)$, more information is required to be gathered to determine their value categories.

6. Sensitivity analysis

In this section, the impact of using diverse parameters α is shown by sensitivity analysis. Each customer manager's preference degree of the distance measure over the GRC is different, so the sensitivity analysis of the parameter α is important. Then we attain the CPs by taking $\alpha = 0, 0.1, 0.2, \ldots, 1$. The CPs with different parameters are presented in Table 9.

Because $D_i^- R_i^- > D_i^+ R_i^+$ for the objects c_3, c_4, c_5, c_8 in the case research, we can find that the CPs increase when the parameters α increase. Because $D_i^- R_i^- < D_i^+ R_i^+$ for the objects c_1, c_2, c_6, c_7 in the case research, we can find that the CPs decrease when the parameters α increase in Table 9 based on Proposition 4.1.

7. Comparison and discussion

Next, we will compare our method with the other methods, and analyze the benefits and restrictions of our method.

7.1. Comparison analyses

To further verify the availability of our method, we compare our method with the other three MCDM methods. The consequences of CPs based on different methods are shown in Table 10 and Fig. 2.

Table 11 shows the ranking consequences obtained by these MCDM methods and the 3WD.

From Table 11, we can learn that the value order of employing the TOPSIS method is $c_5 \succ c_8 \succ c_3 \succ c_4 \succ c_1 \succ c_7 \succ c_6 \succ c_2$, the value order of using the GRA method is $c_5 \succ c_3 \succ c_8 \succ c_4 \succ c_1 \succ c_6 \succ c_7 \succ c_2$, and the value order of using the WA method is $c_5 \succ c_8 \succ c_3 \succ c_4 \succ c_1 \succ c_7 \succ c_6 \succ c_2$. The value orders of the three methods are not completely the same. The value orders are approximately consistent with the consequences of the 3WD model. The customers c_5, c_8, c_3 pertaining to $POS(X)$ are situated in the first three sites in the TOPSIS, GRA, and WA methods. The customers c_7, c_6, c_2 pertaining to $NEG(X)$ are situated in the last three sites. Meanwhile, the customers pertaining to $BND(X)$ are situated in the middle sites of the value order. When $\phi = 0.6, \varphi = 0.4$, the classification results obtained by these methods [20,23,24] are $POS(X) = \{c_5, c_8, c_3\}$; $BND(X) = \{c_4, c_1\}$; $NEG(X) = \{c_7, c_6, c_2\}$. These results verify the validity of our method.

Table 7
The conditional probability of each customer.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
$Pr(X\|c_i)$	0.4494	0.2130	0.7659	0.5251	0.7870	0.2437	0.2446	0.7671

Fig. 2. The consequences of CPs based on different methods.

Table 8
The risk aversion coefficient and threshold values of each customer.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
η_i	0.1011	0.5740	0.5317	0.0501	0.5740	0.5126	0.5108	0.5342
ϕ_i	0.9159	0.7328	0.2121	0.9449	0.1672	0.7469	0.7473	0.2093
φ_i	0.1211	0.8328	0.2577	0.0455	0.2672	0.7655	0.7633	0.2583
χ_i	0.5506	0.7870	0.2341	0.4749	0.2130	0.7563	0.7554	0.2329

7.2. Simulation calculation

This paper chooses the customers $c_2, c_3, c_5, c_6, c_7, c_8$ as the research objects. To test the steadiness of the consequences, two research objects c_1, c_4 are deleted. Then the CPs of the remanent objects are attained by three MCDM methods and our method. The simulation consequences of CPs are shown in Table 12.

The simulation ranking consequences attained by these MCDM methods and the 3WD are shown in Table 13.

Comparing the simulation consequences in Tables 12 and 13 with the calculation consequences in Tables 10 and 11, we can find that they are consistent, which verifies the steadiness of our method.

7.3. Discussions on the advantages and limitations

Based on comparative research, we can learn that the description of the decision result is different, but the decision result attained by the MCDM methods and the 3WD model is roughly the same. The main reason can be attained from the idea of the 3WD. 3WD classifies the customers into three classes: $POS(X)$, $BND(X)$, and $NEG(X)$. These classes are parallel to the high-value, uncertain-value, and low-value customers in the ranking consequences of MCDM methods separately, which in accordance with the decision-making habit of humans.

The principal benefits of our method are shown below:

(1) We employ the 3WDs method in the segmentation of the customers. This method can offer theoretic assist for CRM, and the segmentation consequences are helpful to distribute scarce resources availably, enhance the connection with customers, and obtain realistic competitive superiorities.

(2) It conforms to people's expression habits to evaluate information in the 3WD by DHLTs. The experts can give DHLTs to express criteria information easily and quickly, which can decrease classification cost and economize classification time. We take the relative proximity as the CP which makes segmentation more objective.

However, our method also has some restrictions:

To ease the computation, this paper does not study large-scale data. And we will extend this method to large-scale object classification. Then this method will be more practical.

7.4. Recommendations

The guiding ideology of CRM is to change from the traditional product-centric concept to the modern customer-centric concept, complete grasp of customer information, accurate grasp of customer requirements, rapid response to personalized needs, providing convenient

Table 9
The CPs with different parameters.

	$Pr(X\|c_1)$	$Pr(X\|c_2)$	$Pr(X\|c_3)$	$Pr(X\|c_4)$	$Pr(X\|c_5)$	$Pr(X\|c_6)$	$Pr(X\|c_7)$	$Pr(X\|c_8)$
$\alpha = 0$	0.4690	0.3033	0.6885	0.5227	0.7019	0.3265	0.3242	0.6882
$\alpha = 0.1$	0.4655	0.2879	0.7019	0.5231	0.7165	0.3122	0.3105	0.7018
$\alpha = 0.2$	0.4618	0.2713	0.7161	0.5236	0.7322	0.2969	0.2957	0.7164
$\alpha = 0.3$	0.4579	0.2534	0.7315	0.5241	0.7490	0.2804	0.2800	0.7321
$\alpha = 0.4$	0.4538	0.2340	0.7480	0.5245	0.7673	0.2628	0.2630	0.7489
$\alpha = 0.5$	0.4494	0.2130	0.7659	0.5251	0.7870	0.2437	0.2446	0.7671
$\alpha = 0.6$	0.4449	0.1900	0.7852	0.5256	0.8085	0.2231	0.2247	0.7867
$\alpha = 0.7$	0.4400	0.1649	0.8062	0.5262	0.8319	0.2007	0.2031	0.8080
$\alpha = 0.8$	0.4349	0.1373	0.8291	0.5268	0.8575	0.1763	0.1796	0.8312
$\alpha = 0.9$	0.4294	0.1068	0.8541	0.5274	0.8857	0.1497	0.1538	0.8565
$\alpha = 1.0$	0.4237	0.0730	0.8817	0.5281	0.9168	0.1205	0.1255	0.8842

Table 10
The CPs based on different methods.

Method	$Pr(X\|c_1)$	$Pr(X\|c_2)$	$Pr(X\|c_3)$	$Pr(X\|c_4)$	$Pr(X\|c_5)$	$Pr(X\|c_6)$	$Pr(X\|c_7)$	$Pr(X\|c_8)$
TOPSIS [20]	0.4237	0.0730	0.8817	0.5281	0.9168	0.1205	0.1255	0.8842
GRA [23]	0.4690	0.3033	0.6885	0.5227	0.7019	0.3265	0.3242	0.6881
WA [24]	0.4676	0.1774	0.8466	0.5540	0.8756	0.2167	0.2208	0.8487
Our method	0.4494	0.2130	0.7659	0.5251	0.7870	0.2437	0.2446	0.7671

Table 11
The ranking of the objects based on the CPs.

Method	Ranking
TOPSIS [20]	$c_5 \succ c_8 \succ c_3 \succ c_4 \succ c_1 \succ c_7 \succ c_6 \succ c_2$
GRA [23]	$c_5 \succ c_3 \succ c_8 \succ c_4 \succ c_1 \succ c_6 \succ c_7 \succ c_2$
WA [24]	$c_5 \succ c_8 \succ c_3 \succ c_4 \succ c_1 \succ c_7 \succ c_6 \succ c_2$
Our method	$POS(X) = \{c_5, c_8, c_3\}; BND(X) = \{c_4, c_1\}; NEG(X) = \{c_7, c_6, c_2\}$

Table 12
The simulation consequences of CPs based on different methods.

Method	$Pr(X\|c_2)$	$Pr(X\|c_3)$	$Pr(X\|c_5)$	$Pr(X\|c_6)$	$Pr(X\|c_7)$	$Pr(X\|c_8)$
TOPSIS [20]	0.0730	0.8817	0.9168	0.1205	0.1255	0.8842
GRA method [23]	0.3033	0.6885	0.7019	0.3265	0.3242	0.6881
WA [24]	0.1774	0.8466	0.8756	0.2167	0.2208	0.8487
Our method	0.2130	0.7659	0.7870	0.2437	0.2446	0.7671

Table 13
The simulation ranking of the objects based on CPs.

Method	Ranking
TOPSIS [20]	$c_5 \succ c_8 \succ c_3 \succ c_7 \succ c_6 \succ c_2$
GRA [23]	$c_5 \succ c_3 \succ c_8 \succ c_6 \succ c_7 \succ c_2$
WA [24]	$c_5 \succ c_8 \succ c_3 \succ c_7 \succ c_6 \succ c_2$
Our method	$POS(X) = \{c_5, c_8, c_3\}; NEG(X) = \{c_7, c_6, c_2\}$

purchase channels, and regular customer care. The core problem for enterprises to correctly implement CRM is to take effective methods to classify their customers reasonably, find customer value, concentrate limited resources on high-value customers, provide better services for them, and retain high-value customers to prevent their loss. And through the classification to establish the corresponding customer service system and implement differentiated customer service management. Correct and reasonable customer classification enables enterprises to adopt different marketing strategies for customers with different values, and rationally allocate corporate resources to maximize corporate benefits.

Based on the management strategy, for high-value customers, such as c_5, c_8, c_3, the enterprise should concentrate resources to focus on maintaining these customers, and try to maintain their loyalty to the company with quality service. For low-value customers, such as c_7, c_6, c_2, the enterprise does not need to invest too many resources to maintain. For the other customers located in $BND(X)$, more information is required to be gathered to determine their value categories.

8. Conclusions

In this research, we discuss the segmentation problem of customers under DHLT information and employ the 3WD method to settle this problem. First, the criteria are evaluated by using the DHLT which conforms to people's expression habits. The weights of criteria are attained according to the maximum deviation method, which is one of the objective weighting methods. Second, the CPs are estimated by the relative proximities which are calculated by the improved TOPSIS method. The improved TOPSIS method considers both distance measure and gray relation degree. The threshold values are calculated by the relative utilities which are constructed on the basis of the criteria information. The relative proximities can be used to calculate the CPs and the thresholds objectively, which makes the classification result more scientific and reasonable. Finally, the segmentation of customers is derived according to the maximum-utility principle, and the corresponding CRM recommendations are given, then the enterprises can obtain maximum returns with minimum investment. And the steadiness of our method has been verified by the simulation consequences. In the future, we will attend to extend this method to large-scale object classification.

CRediT authorship contribution statement

Xiang Li: Writing – original draft, Software, Methodology, Conceptualization. **Zeshui Xu:** Writing – review & editing, Investigation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

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